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A MONTE CARLO STUDY OF THE USE OF AUXILIARY INFORMATION IN THE --ETC(U)

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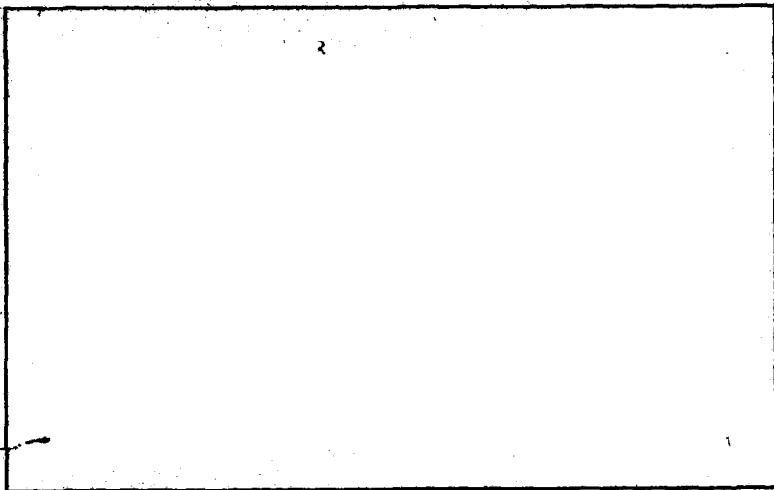
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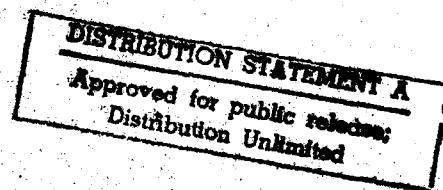
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Applied Research in Statistics - Mathematics - Operations Research

(6) A MONTE CARLO STUDY OF  
THE USE OF AUXILIARY INFORMATION  
IN THE DEVELOPMENT OF AN IMPACT  
ACCELERATION INJURY PREDICTION MODEL.

by

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## I. INTRODUCTION

As a major part of its impact acceleration research program, the Naval Biodynamics Laboratory is investigating the dynamic response of the head/neck system as a function of motion and anthropometric parameters. One goal of this program is the development of an impact acceleration injury prediction model which permits reliable inferences about injury probability as a function of dynamic and physical variables. Two previous Desmatics reports [2,3] have discussed procedures for incorporating various sources of auxiliary information into a model.

This report describes a Monte Carlo investigation of the procedure for using preinjury "side effect" data based on evoked potential response. The notation and discussion in this report is a continuation of that of the previous reports. Thus, the reader may find a review of those reports helpful as a background.

## II. THE MONTE CARLO STUDY

In order to assess the performance of the procedure for relatively small samples, 500 Monte Carlo simulations were performed using a sample size of 25 observations. The goal of this Monte Carlo investigation was to obtain an indication of the effect of the size of the correlation coefficient,  $\rho$ , between the injury tolerance and the preinjury side effects on the mean square error (MSE) of the weighted least squares estimates of  $\beta$ . Furthermore, this Monte Carlo study assumed no prior information linking the parameters of  $\beta_1$  and  $\beta_2$  in order to impose a more stringent test of the contribution of the empirical auxiliary information to decreasing the MSE's of the probit estimates of the elements in  $\beta_1$  when the number of observations is small.

### A. PROCEDURE

Root mean square error (RMSE), the square root of MSE, was used as the basis for performance evaluation. RMSE's of two different weighted least squares estimates of  $\beta_1$  were compared for five values of  $\rho$ : -0.1, -0.3, -0.5, -0.7, -0.9. The first estimate of  $\beta_1$  was derived from a probit model that is not conditional on the linear regression residuals. This is referred to as the "standard" model. The second estimate of  $\beta_1$  was derived from a probit model that is conditional on the linear regression residuals. This model is referred to as the "modified" model.

The "standard" and "modified" probit models and the linear regression model all have the same predictor variable. Values for the predictor variable are sampled from a uniform distribution between -1 and 1. For each simulation trial, a prior unbiased estimate of  $\beta_{11}$  was sampled from a normal distribution with mean  $\beta_{11}$  and standard deviation 0.5. This estimate of  $\beta_{11}$  was incorporated into both the "standard" and "modified" probit models as outlined in [2].

Thus, the information employing the "standard" model can be represented by

$$r = \beta_{11} + v$$

$$y_{1i} = \Phi(\beta_{01} + \beta_{11}x_i) + \varepsilon_{1i}, \quad i = 1, \dots, 25$$

where  $E(v) = 0$ ,  $Var(v) = 0.5$ ,

$$E(\varepsilon_{1i}) = 0, \quad Var(\varepsilon_{1i}) = p_i(1-p_i), \quad p_i = \Phi(\beta_{01} + \beta_{11}x_i),$$

and  $r$  is an unbiased estimate of  $\beta_{11}$ .

Similarly, the information conditional on the linear regression residuals can be represented using the "modified" probit model by

$$y_{1i} = \Phi[(1-\rho^2)^{-\frac{1}{2}} (\beta_{01} + \beta_{11}x_i - \rho s_i)] + \xi_i, \quad i=1, \dots, 25$$

where  $E(\xi_i) = 0$ ,  $Var(\xi_i) = p_i(1-p_i)$ ,  $p_i = \Phi[(1-\rho^2)^{-\frac{1}{2}} (\beta_{01} + \beta_{11}x_i - \rho s_i)]$

and  $s_i$  is the  $i^{\text{th}}$  standardized, maximum likelihood, linear regression residual.

The parameters of the linear regression model used in the simulation were  $\beta_{01} = \beta_{11} = \sigma_2 = 1.0$ . The starting values for the probit iterations

for  $\beta_{01}$ ,  $\beta_{11}$ , and  $\rho$  were 0.5, 0.5, and -0.5, respectively.

## B. RESULTS

Figure 1 contains the Monte Carlo estimates of the means, standard deviations, RMSE's and asymptotic standard deviations for the estimates of  $\beta_{01}$  and  $\beta_{11}$  for both the "standard" and "modified" models. Figure 1 also lists the Monte Carlo estimates of the ratio of the corresponding RMSE's of the "standard" model to the "modified" model for both  $\beta_{01}$  and  $\beta_{11}$ . Figure 2 lists the Monte Carlo estimates of the mean, standard deviation, RMSE and asymptotic standard deviation for each estimate of  $\rho$ .

For labeling purposes, estimates of  $\beta_{01}$  and  $\beta_{11}$  from the "standard" model will be denoted by  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$ , whereas estimates of  $\beta_{01}$  and  $\beta_{11}$  from the "modified" model will be denoted by  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$ . As can be seen from Figure 1,  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$  each have smaller estimated RMSE's than their counterparts  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$ . (However, the bias of  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$  is slightly larger than the bias for  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$ , respectively.) Increases in the absolute value of  $\rho$  tend to reduce the corresponding RMSE's of  $\hat{\beta}_{01}$  and  $\hat{\beta}_{11}$ , with the tendency more pronounced for  $\hat{\beta}_{11}$ .

Except for the estimate of  $\hat{\beta}_{11}$  for  $\rho = -0.7$ , the average asymptotic standard deviations are smaller than the corresponding average estimated standard deviations derived from the 500 Monte Carlo trials. This is true for both the "standard" and "modified" models.

Figure 2 shows the Monte Carlo results for the estimation of  $\rho$  via the "modified" model. The average estimated  $\rho$  has a bias of roughly a little over 0.1. It is therefore suggested that the approximate unbiased estimate of  $\rho$  as discussed in [3] also be computed in addition to the

$\rho$	Parameter	Parameter Estimate				Standard Deviation		RMSE		Asymptotic Standard Deviation	
		Standard Model	Modified Model	Standard Model	Modified Model	Standard Model	Modified Model	Ratio of RMSE's	Standard Model	Modified Model	
-0.1	$\beta_{01} = 0.0$	-0.0009	-0.0103	0.2894	0.2855	0.2894	0.2857	0.9872	0.2745	0.2665	
-0.3	$\beta_{01} = 0.0$	-0.0045	-0.0113	0.2745	0.2733	0.2745	0.2735	0.9964	0.2738	0.2635	
-0.5	$\beta_{01} = 0.0$	0.0139	0.0196	0.2805	0.2779	0.2808	0.2786	0.9922	0.2741	0.2556	
-0.7	$\beta_{01} = 0.0$	0.0137	0.0312	0.2768	0.2724	0.2771	0.2742	0.9895	0.2740	0.2401	
-0.9	$\beta_{01} = 0.0$	0.0025	0.0314	0.2858	0.2690	0.2858	0.2708	0.9475	0.2738	0.2177	
<hr/>											
-0.1	$\beta_{11} = 1.0$	0.9962	0.9610	0.3779	0.3654	0.3779	0.3675	0.9725	0.3565	0.3509	
-0.3	$\beta_{11} = 1.0$	0.9904	0.9512	0.3661	0.3446	0.3662	0.3480	0.9503	0.3559	0.3486	
-0.5	$\beta_{11} = 1.0$	0.9960	0.9464	0.3699	0.3424	0.3699	0.3466	0.9370	0.3563	0.3427	
-0.7	$\beta_{11} = 1.0$	0.9949	0.9449	0.3597	0.3273	0.3597	0.3319	0.9227	0.3562	0.3313	
-0.9	$\beta_{11} = 1.0$	0.9822	0.9414	0.3605	0.3213	0.3609	0.3266	0.9050	0.3558	0.3124	

Figure 1: Monte Carlo Estimates for  $\beta_{01}$  (intercept parameter) and  $\beta_{11}$  (slope parameter)  
Based on 500 Simulations.

<u>Parameter</u>	<u>Parameter Estimate</u>	<u>Standard Deviation</u>	<u>RMSE</u>	<u>Asymptotic Standard Deviation</u>
$\rho = -0.1$	0.0062	0.2917	0.3064	0.2497
$\rho = -0.3$	-0.1864	0.2754	0.2979	0.2429
$\rho = -0.5$	-0.3741	0.2422	0.2730	0.2213
$\rho = -0.7$	-0.5757	0.1836	0.2217	0.1820
$\rho = -0.9$	-0.7554	0.1333	0.1967	0.1319

Figure 2: Monte Carlo Estimates for  $\rho$  (correlation coefficient) Based on 500 Simulations.

estimate of  $\rho$  from the "modified" model. Note that the RMSE of the estimate of  $\rho$  tends to decrease with increasing absolute values of  $\rho$  and that the estimated asymptotic standard deviation is always smaller than the estimated standard deviation.

#### C. SUMMARY

Auxiliary empirical information based on a preinjury "side effect" measurement should be helpful in reducing the MSE of the probit prediction model parameter estimates in the estimation situation presented in the previous section. Even though the sample size was only 25 and there was no prior information linking  $\underline{\beta}_1$  and  $\underline{\beta}_2$ , the auxiliary empirical information noticeably reduced the MSE of the probit estimate of  $\underline{\beta}_1$ . Had there been prior information linking  $\underline{\beta}_1$  and  $\underline{\beta}_2$ , e.g.,  $\beta_{11} = \beta_{12}/\sigma_2$ , a much larger reduction in MSE would have taken place. Fortunately, this parameter constraint situation as discussed in [3] seems reasonable. Therefore, further consideration of the existence of such a constraint should be taken into account.

### III. DISCUSSION

An important statistical topic, somewhat connected with the simultaneous analysis of the injury and preinjury data, is the subject of "seemingly unrelated" regression estimation. (See [1], for example.) By "seemingly unrelated" regression estimation, it is meant that two or more regression models are simultaneously estimated under the assumption that the error terms are correlated within each overall observation. When the regression models are linear, then some important facts are known. If the design matrices of the linear regression models are all equal, then the regression estimates are essentially the same whether or not they are obtained from separate regressions or from one simultaneous regression. On the other hand, if the intercorrelations between the predictor variables in the design matrices are low, then the regression coefficients estimated simultaneously have much lower variances than the regression coefficients estimated from the separate regression models, provided that the correlations within each vector of error terms are high.

The usual impact acceleration experiment, however, has the same predictor variables for the preinjury as well as for the injury model. The only reason that the probit estimates of  $\beta_1$  from the "standard" model are different from the probit estimates of  $\beta_1$  from the "modified" model is that the functional form of the probit model is different from the linear preinjury model. It is suspected though, that considerable improvement in the probit estimate of  $\beta_1$  could be achieved if  $\rho$  were large in absolute value and simultaneously the predictor variables for the probit model

had low correlations with the predictor variables for the linear preinjury model.

The substantiation of this conjecture would require another Monte Carlo investigation. Furthermore, the feasibility of such an experimental design needs to be considered. If the conjecture holds true, then an experimental design could be constructed to significantly improve the probit estimates of  $\beta_1$ . This might be done by first running all test subjects at acceleration levels that are low enough not to cause injury, but that are high enough to give informative preinjury readings. "Center" these readings by subtracting their average values and dividing by their estimated standard deviations. Next try to run subjects at various low, intermediate, and high acceleration levels such that when these levels are "centered" and paired, by subject, with the low preinjury "centered" levels a pair of orthogonal matrices results.

For example, if the centered design matrix for the subjects run at low, preinjury levels is given by  $\underline{X}_1$  and the centered design matrix for subjects run at various low, intermediate, and high acceleration levels is given by  $\underline{X}_2$  then  $\underline{X}_1' \underline{X}_2 = \underline{0}$ . This type of experimental design should have the property to decrease the variances of the probit estimates for every decrease in  $\rho$ . Furthermore, moderate to high negative values of  $\rho$  should result in substantial decreases in the MSE of the probit estimates obtained simultaneously using the preinjury data compared with those obtained from the "standard" model.

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